

A Pythagorean Fuzzy-Based MUNRA Method for Handling Uncertainty in Complex Decision Environments

Setiawansyah

Informatics, Universitas Teknokrat Indonesia, Indonesia
setiawansyah@teknokrat.ac.id

Abstract: This research develops the Pythagorean Fuzzy Multi-Normalized Rating Analysis (PF-MUNRA) method as a novel approach to address uncertainty and ambiguity in multi-criteria decision making. The main contribution of this study lies in the integration of Pythagorean Fuzzy Sets with a multi-normalization framework consisting of linear, vector, and non-linear normalization within a single decision-making model, enabling more flexible, comprehensive, and unbiased evaluation results compared to conventional single-normalization approaches. This method integrates the concept of Pythagorean Fuzzy Sets, which can represent degrees of membership and non-membership more flexibly, with the multi-normalization approach in MUNRA. Unlike previous studies that generally apply fuzzy environments and normalization techniques separately, the proposed PF-MUNRA simultaneously combines fuzzy uncertainty handling, multi-normalization mechanisms, and objective weighting to improve ranking consistency and decision robustness. In addition, weighted aggregation is used to produce more accurate preference values and reflect the relative importance of each criterion. The experimental results demonstrate that PF-MUNRA produces stable alternative rankings with Spearman correlation values ranging from 0.9464 to 1.0000 under various weight-change scenarios, indicating a very strong level of ranking consistency and robustness. Comparative analysis shows changes in alternative positions that reflect the capability of the proposed method to capture data complexity more effectively than the initial approach, while sensitivity analysis confirms that variations in criterion weights do not significantly affect the final ranking results, thereby proving that PF-MUNRA has high stability and reliability in dynamic and uncertain decision-making environments.

Keywords: Pythagorean Fuzzy Sets; PF-MUNRA; Multi-Criteria Decision Making; Multi-normalization; Sensitivity Analysis;

1. INTRODUCING

Decision-making in complex environments is often faced with uncertainty, ambiguity, and incomplete information, making the process of determining the best option increasingly challenging[1], [2]. In such situations, decision-makers are not only required to understand the various available alternatives but also must be able to evaluate many criteria that are often conflicting. Uncertainty arising from data limitations can result in less accurate decision outcomes and potentially increase risks. Furthermore, ambiguity in information interpretation can also trigger differences in perception that impact inconsistency in decision-making[3], [4]. Therefore, a systematic and structured approach is needed so that the evaluation process can be carried out more objectively and

comprehensively, enabling the resulting decisions to reflect actual conditions and minimize potential errors.

The multi-criteria decision making (MCDM) method is widely used in various fields to assist decision-making processes that involve multiple criteria, because it is able to provide a systematic framework for evaluating alternatives[5]–[7]. However, classical approaches in MCDM often have limitations in capturing the uncertainty inherent in data and decision-makers' preferences. This is because most traditional methods assume that the data is certain and can be represented deterministically, whereas in practice, the information available is often incomplete or contains ambiguities[8]–[10]. As a result, the obtained outcomes may be less accurate and not fully reflect the complex real conditions. Therefore, it is necessary to develop or integrate more adaptive approaches, such as the use of fuzzy concepts, probabilistic methods, or data distribution-based techniques, so that uncertainty can be modeled more effectively and produce more reliable decisions[11]–[13].

The theory of Pythagorean Fuzzy Sets (PFS) offers greater flexibility compared to traditional fuzzy approaches because it is able to represent uncertainty more broadly through the independent measurement of membership and non-membership degrees[14], [15]. In this concept, both degrees are not only considered separately, but are also constrained by the condition of the sum of their squares, which provides a greater tolerance space compared to previous methods such as Intuitionistic Fuzzy Sets[11], [16], [17]. This allows PFS to capture more complex and ambiguous information, especially in situations where the level of doubt or uncertainty is quite high. With this flexibility, PFS becomes more effective in handling uncertain and incomplete data, thus capable of improving the quality of decision-making results[18], [19]. The application of PFS within the MCDM framework becomes a promising solution to produce decisions that are more accurate, adaptive, and representative of real conditions[20]–[22].

The multi-normalized rating analysis (MUNRA) method is known to be capable of improving accuracy in the decision-making process by utilizing multiple normalization techniques simultaneously[23]. This approach allows data from various scales and characteristics to be standardized more comprehensively, resulting in a more balanced evaluation that is not biased toward any particular normalization method. By combining multiple normalization schemes, MUNRA can reduce information distortion that often occurs in single-method approaches, especially when the data has high variation or an uneven distribution. Additionally, this method also helps enhance the stability of ranking results by considering various perspectives in the alternative evaluation process. Therefore, MUNRA becomes one of the effective approaches within the MCDM framework to produce more robust, consistent, and reliable decisions. However, classical MUNRA is still based on crisp data, making it less optimal when faced with conditions of high uncertainty. This approach assumes that all data used is certain and clearly defined, whereas in many real-world cases the available information is often ambiguous, incomplete, or contains subjectivity. As a result, the ability of this method to represent the complexity of situations becomes limited, so the decision outcomes may be less accurate and less reflective of the actual conditions. In addition, the use of crisp data also makes MUNRA less flexible in accommodating decision-makers' preferences that are linguistic or perception-based. Therefore, further development is needed, such as integration with fuzzy approaches, so that this method can work more adaptively and effectively in environments full of uncertainty[24]–[26].

The MUNRA method in previous research has not been fully optimal because, although it has accommodated several normalization techniques, the approach used is still not completely able to eliminate the bias that arises from dependence on data scales and alternative distribution characteristics, causing ranking results to differ significantly when the data structure changes. In addition, a specific weakness of MUNRA lies in the absence of a strong mechanism to handle uncertainty and ambiguity in decision-making data, so

the evaluation values remain deterministic and do not adequately represent real conditions, which are often uncertain. Previous research also showed that MUNRA is still quite sensitive to changes in criteria weights, which can cause instability in the ranking of alternatives when there are variations in decision-makers' preferences. To address this issue, PF-MUNRA was developed by integrating Pythagorean Fuzzy Sets, which allow a more flexible representation of membership and non-membership degrees in handling uncertainty, as well as combining a multi-normalization approach (linear, vector, and non-linear) to reduce dependence on a specific normalization method. With this integration, PF-MUNRA is able to produce a more adaptive, stable, and robust evaluation process, and improve the consistency of ranking results even in the presence of data variations or changes in criteria weights.

Therefore, the development of the Pythagorean Fuzzy MUNRA (PF-MUNRA) method is needed to overcome the limitations of the classical approach in dealing with uncertain and linguistic data. The integration of the Pythagorean Fuzzy Sets concept into the MUNRA framework allows for a more flexible representation of information through membership and non-membership degrees that can capture the level of hesitation more accurately. With this approach, data that was previously difficult to measure precisely can be modeled more realistically according to actual conditions in the field. In addition, PF-MUNRA still retains the advantages of MUNRA in the use of multi-normalization, resulting in more stable and comprehensive evaluations. This combination makes the PF-MUNRA method more adaptive in handling the complexity of decision-making, especially in situations involving high uncertainty and subjective preferences. Thus, this method has the potential to provide decision outcomes that are more accurate, consistent, and representative.

The integration of Pythagorean Fuzzy with the MUNRA method is still very limited in the literature, so the development of approaches capable of accommodating uncertainty more comprehensively has not been widely explored. This limitation indicates that most research still focuses on conventional approaches without utilizing the advantages of a more flexible fuzzy representation. Furthermore, there is still a lack of approaches that can effectively combine flexibility in handling ambiguous data through the fuzzy concept with the stability of evaluation results produced by the multi-normalization technique in MUNRA. In fact, the combination of these two aspects is very important to produce an adaptive and consistent decision-making system. Furthermore, research that specifically tests the effectiveness of the PF-MUNRA method in complex decision-making environments is still relatively limited, so there is not yet enough strong empirical evidence regarding the advantages and performance of this method. Therefore, further research is needed to fill this gap while also strengthening validation of the application of PF-MUNRA in various dynamic and uncertainty-filled decision contexts.

This study aims to develop the PF-MUNRA method as an approach capable of handling uncertainty in the decision-making process more effectively. This development is carried out by integrating the concept of Pythagorean Fuzzy Sets into the MUNRA framework, so that ambiguous and linguistic data can be represented more flexibly and realistically. In addition, this study also adapts the normalization process in MUNRA into the Pythagorean fuzzy domain, with the aim of maintaining the advantages of multi-normalization while enhancing the method's ability to manage uncertain data variations. Through this adaptation, it is expected that the alternative evaluation process will become more stable and less sensitive to data changes. Furthermore, the performance of the PF-MUNRA method is evaluated to assess its ability to produce accurate, consistent, and reliable alternative rankings. This research can contribute to the development of decision-making methods that are more adaptive to the complexity and uncertainty of the real environment.

2. RESEARCH METHODOLOGY

Research methods are a series of systematic steps used to obtain, process, and analyze data in order to achieve research objectives in a structured and scientific manner[27], [28]. In the context of decision-making, this method plays an important role in ensuring that each stage is carried out logically, objectively, and can be justified. Through the application of appropriate methods, researchers can identify problems, determine suitable approaches, and produce solutions that are relevant to the conditions faced. In addition, research methods also help reduce bias and improve the accuracy of the results obtained.

Research Stage

The research stages are a sequence of steps systematically arranged to conduct a study from beginning to end. These stages reflect a structured workflow so that the research process can proceed in a directed, logical, and consistent manner. Each stage is interconnected and designed to ensure that the research objectives can be achieved effectively. In addition, having research stages helps maintain the clarity of the process and enhances the reliability of the results obtained. Thus, the research stages become an important part in ensuring the quality and validity of a study, as shown in Figure 1.

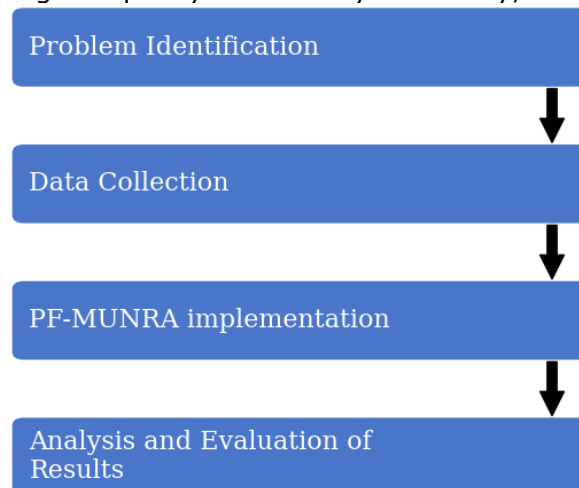


Figure 1. Research Stage

This research stage begins with problem identification to clearly understand the context, objectives, and scope of the problem to be solved. Next, relevant data is collected as a basis for the decision-making process, whether in the form of numerical or linguistic data that reflects real conditions. The following stage is the implementation of the PF-MUNRA method, which includes transforming data into Pythagorean fuzzy form, multi-technique normalization processes, and calculating preference values for each alternative. After all calculations are completed, the final stage is the analysis and evaluation of the results to assess the method's performance in producing accurate, consistent rankings that can represent complex decision-making conditions.

Pythagorean Fuzzy MUNRA (PF-MUNRA)

The PF-MUNRA method was developed as a new approach in multi-criteria decision making to address the limitations of conventional methods in handling uncertainty and variations in data distribution. This method integrates the concept of Pythagorean fuzzy, which represents degrees of membership and non-membership more flexibly, with a multiple normalization approach involving linear, vector, and non-linear techniques. This combination allows for a more comprehensive and stable evaluation of alternatives, as it

considers various normalization perspectives simultaneously. Thus, PF-MUNRA is able to produce more accurate and reliable preference values in supporting complex decision-making processes.

The first stage in the PF-MUNRA method is to construct a decision matrix based on Pythagorean fuzzy, which is used to represent the assessment level of alternatives against each criterion under conditions of uncertainty using (1)

$$X = [x_{ij}]_{m \times n} \quad (1)$$

Each element of the matrix is represented as a pair in the following form.

$$x_{ij} = (\mu_{ij}, v_{ij}) \quad (2)$$

where

$$\mu_{ij}^2 + v_{ij}^2 \leq 1 \quad (3)$$

Matrix X represents a decision matrix with m alternatives and n criteria, where x_{ij} is the evaluation value of the i^{th} alternative against the j^{th} criterion. The symbol $x_{ij} = (\mu_{ij}, v_{ij})$ is a Pythagorean fuzzy number consisting of a membership degree μ_{ij} and a non-membership degree v_{ij} ; and $\mu_{ij}^2 + v_{ij}^2 \leq 1$ is a validity constraint in Pythagorean Fuzzy Sets.

The second stage is to normalize the decision matrix using three different approaches, namely linear, vector, and non-linear. Normalization values are calculated using the following equation.

$$r_{ij}^{\mu} = \begin{cases} \frac{x_{ij}^{\mu}}{\max x_{ij}^{\mu}}; \text{benefit} \\ \frac{\min x_{ij}^{\mu}}{x_{ij}^{\mu}}; \text{cost} \end{cases} \quad (4)$$

$$r_{ij}^{v} = \begin{cases} \frac{x_{ij}^{v}}{\max x_{ij}^{v}}; \text{benefit} \\ \frac{\min x_{ij}^{v}}{x_{ij}^{v}}; \text{cost} \end{cases} \quad (5)$$

$$y_{ij}^{\mu} = \begin{cases} \frac{x_{ij}^{\mu}}{\sqrt{\sum_{i=1}^m (x_{ij}^{\mu})^2}}; \text{benefit} \\ 1 - \frac{x_{ij}^{\mu}}{\sqrt{\sum_{i=1}^m (x_{ij}^{\mu})^2}}; \text{cost} \end{cases} \quad (6)$$

$$y_{ij}^{v} = \begin{cases} \frac{x_{ij}^{v}}{\sqrt{\sum_{i=1}^m (x_{ij}^{v})^2}}; \text{benefit} \\ 1 - \frac{x_{ij}^{v}}{\sqrt{\sum_{i=1}^m (x_{ij}^{v})^2}}; \text{cost} \end{cases} \quad (7)$$

$$p_{ij}^{\mu} = \begin{cases} \left(\frac{x_{ij}^{\mu}}{\max x_{ij}^{\mu}}\right)^2; \text{benefit} \\ \left(\frac{\min x_{ij}^{\mu}}{x_{ij}^{\mu}}\right)^3; \text{cost} \end{cases} \quad (8)$$

$$p_{ij}^{v} = \begin{cases} \left(\frac{x_{ij}^{v}}{\max x_{ij}^{v}}\right)^2; \text{benefit} \\ \left(\frac{\min x_{ij}^{v}}{x_{ij}^{v}}\right)^3; \text{cost} \end{cases} \quad (9)$$

The symbols r_{ij}^{μ} and r_{ij}^{v} represent linear normalization processes for membership and non-membership degree values based on benefit and cost criteria, the symbols y_{ij}^{μ} and y_{ij}^{v} represent vector normalization using the square root of the sum of squares (Euclidean norm) to standardize the data, while the symbols p_{ij}^{μ} and p_{ij}^{v} represent a form of non-linear normalization using power transformation (square for benefit and cube for cost) to enhance differences in values between alternatives in the evaluation process.

The third stage is to aggregate the normalization results by considering the weight of each criterion. The weights reflect the relative importance level of each criterion in the decision-making process. The aggregate value is calculated using the following equation.

$$r_i' = \sum_{j=1}^n w_j * (r_{ij}^{\mu} - r_{ij}^{\nu}) \quad (10)$$

$$y_i' = \sum_{j=1}^n w_j * (y_{ij}^{\mu} - y_{ij}^{\nu}) \quad (11)$$

$$p_i' = \sum_{j=1}^n w_j * (p_{ij}^{\mu} - p_{ij}^{\nu}) \quad (12)$$

The symbol r_i' , y_i' , and p_i' represents the final aggregate value for each normalization approach (linear, vector, and non-linear), obtained by multiplying the criterion weights w_j by the difference between the membership and non-membership degrees ($\mu - \nu$) for each criterion, and then summing them across all n criteria, thus resulting in the total preference value for each alternative.

The final stage is to integrate all aggregation results into a single final value using the basic concept of MUNRA. The final score is calculated using the following equation.

$$S_i = \alpha r_i' + \beta y_i' + \gamma p_i' \quad (13)$$

The S_i symbol is the final score value of each alternative obtained from a weighted combination of the results of three normalization approaches, with parameters α , β , and γ as the contribution coefficients of each method, thus producing a final preference value that represents the overall performance of the alternatives in the decision-making process.

The proposed PF-MUNRA method effectively integrates Pythagorean fuzzy representation with multiple normalization techniques to enhance decision-making under uncertainty. The method provides stable, consistent, and reliable ranking results, making it suitable for complex multi-criteria decision environments.

3. RESULT AND DISCUSSIONS

The Pythagorean fuzzy-based MUNRA method was developed as an advanced approach capable of handling uncertainty, vagueness, and ambiguity in complex decision-making environments, particularly when the available data is subjective, incomplete, or inconsistent. By utilizing the concept of Pythagorean fuzzy sets, this method offers greater flexibility than conventional fuzzy approaches in representing membership and non-membership degrees, allowing the decision maker's hesitation level to be modeled more accurately and realistically. This capability is highly important in MCDM problems where human judgment and qualitative assessments often dominate the evaluation process.

The integration of the Pythagorean fuzzy environment with the MUNRA framework enables a systematic aggregation of utility values through normalization, weighting, and comprehensive ranking of alternatives. In addition, this method can effectively balance beneficial and non-beneficial criteria while maintaining consistency in the evaluation process. The resulting decision outcomes are more stable, adaptive, and reliable because they reflect real-world conditions that are often characterized by uncertainty and dynamic changes. Therefore, the Pythagorean fuzzy-based MUNRA method is considered highly suitable for solving various practical MCDM problems in fields such as supplier selection, employee evaluation, healthcare management, financial assessment, and other strategic decision-making applications.

Problem Identification

The identification of problems in the application of the PF-MUNRA method arises from the complexity of the modern decision-making environment, which is characterized by many criteria, diverse alternatives, as well as data that is not always complete, certain, or consistent. In MCDM, decision-makers are often faced with information that is subjective, ambiguous, or even contradictory, making it difficult to represent accurately using conventional deterministic or fuzzy approaches. In addition, the interdependence among criteria and differences in perception among evaluators further increase the level of

uncertainty. This problem becomes even more complex when the value aggregation process is carried out directly without considering the level of doubt or inconsistency in the assessment, which ultimately can result in biased decisions that do not accurately reflect the actual conditions.

Furthermore, traditional aggregation methods often fail to comprehensively integrate uncertainty information in the alternative ranking process. This causes the final results to tend to be unstable when there are small changes in the criterion weights or evaluation values. On the other hand, the need for methods that can accommodate flexibility in data representation while maintaining consistency in decision results is becoming increasingly important. Therefore, an approach is needed that is not only capable of handling uncertainty through stronger representations such as Pythagorean fuzzy sets, but also able to combine it with systematic aggregation mechanisms like MUNRA. Thus, the identification of this problem emphasizes the importance of developing methods that are more adaptive, robust, and capable of providing reliable ranking results under complex and dynamic decision-making environments.

Data Collection

Data collection in this study uses a dataset from the research [29] consisting of technicians who are evaluated based on several relevant performance criteria, namely technical expertise (C1), timeliness (C2), quality of work results (C3), and efficiency (C4). Each alternative is given a score on each criterion reflecting the technician's performance on the criterion. The assessment data in this study contains evaluations of each technician's performance against a set of predetermined criteria, where each score reflects the level of ability or performance of the technician based on the aspects. These assessments are generally provided in the form of a standardized numerical scale, which facilitates the process of comparison among alternatives, while also ensuring consistency in evaluation. All assessment data used in this process are presented in full in Table 1.

Table 1. Alternative Assessment Data

Alternative	C1	C2	C3	C4
Candidate A	4	5	4	3
Candidate B	3	4	5	4
Candidate C	5	3	4	5
Candidate D	2	5	3	4
Candidate E	4	4	5	5
Candidate F	3	2	4	3
Candidate G	5	3	3	4

The assessment data in Table 1 shows the performance variation of each candidate against the four criteria, where the differences in values among the alternatives for each criterion serve as an important basis for determining the relative importance of these criteria. Through an objective weighting approach using entropy, criteria that have higher value variations among candidates will receive greater weights because they are considered to provide more significant information in the decision-making process, whereas criteria with lower variations tend to have smaller weights. The criteria weights used in this study are presented in Table 2.

Table 2. Criteria Weight

C1	C2	C3	C4
0.3451	0.3451	0.1549	0.1549

Linguistic evaluation for assessment criteria is an approach used to assess the level of importance of a criterion based on the subjective perceptions of decision-makers using linguistic terms, making it easier for experts to provide evaluations when numerical data is difficult to obtain or contains uncertainty[30]. This approach is then converted into a numerical form, such as Pythagorean fuzzy numbers, in order to be processed mathematically in multi-criteria decision-making, without losing the original meaning of human judgment. Linguistic evaluation serves as a bridge between qualitative assessment and systematic quantitative analysis, particularly in dealing with ambiguity and data uncertainty. Table 1 contains the linguistic data used as a reference in the evaluation process, which includes the mapping between linguistic terms and Pythagorean fuzzy values for each level of criterion importance.

Table 3. Linguistic Terms for Rating the Importance of Criteria and Decision Makers

Value	Description	PFNs
5	Excellent	(0.96, 0.27)
4	Good	(0.83, 0.46)
3	Average	(0.65, 0.54)
2	Poor	(0.46, 0.73)
1	Very Poor	(0.38, 0.84)

The importance level of criteria and decision-makers is represented using linguistic terms based on Pythagorean Fuzzy Numbers (PFNs) as shown in Table 3, allowing this approach to represent importance levels more flexibly and to handle uncertainty in the decision-making process. Overall, the data collection process in this study resulted in a structured and consistent dataset, which includes performance assessments of technicians based on relevant criteria and is supported by objective weighting using the entropy method to capture the importance level of each criterion more accurately. In addition, the integration of a linguistic evaluation approach converted into Pythagorean fuzzy numbers provides an additional capability to represent uncertainty and subjectivity in the assessments, so that the obtained data is not only quantitative but also capable of accommodating qualitative aspects. Thus, the dataset used has met the needs of analysis in multi-criteria decision-making and serves as a strong foundation for subsequent processing and evaluation stages.

PF-MUNRA Implementation

PF-MUNRA Implementation is a general stage in the application of multi-criteria decision-making methods used to evaluate and compare several alternatives based on various predetermined criteria. At this stage, data that has been previously collected and prepared is processed through a series of systematic calculation procedures to produce preference values for each alternative. This method integrates the concept of criteria weighting and value comparison between alternatives, thus providing a comprehensive picture of relative performance. The result of this process is a ranking of alternatives that can be used as a basis for determining the best choice according to the decision-making objectives.

The first stage in the PF-MUNRA method is to build a decision matrix based on fuzzy Pythagoras, which is used to represent the level of assessment of alternatives for each criterion under conditions of uncertainty using (1), (2), and (3) based on the assessment data in Table 1 by following the Linguistic terms rules in Table 2.

$$x_{11} = (\mu_{11}, v_{11}) = (0.83, 0.46)$$

$$\mu_{11}^2 + v_{11}^2 \leq 1; 0.83^2 + 0.46^2 \leq 1; 0.9005 \leq 1$$

The converted values of the PF-MUNRA decision matrix are presented in Table 4.

Table 4. Decision matrix of the PF-MUNRA method.

Alternative	C1	C2	C3	C4
Candidate A	(0.83, 0.46)	(0.96, 0.27)	(0.83, 0.46)	(0.65, 0.54)
Candidate B	(0.65, 0.54)	(0.83, 0.46)	(0.96, 0.27)	(0.83, 0.46)
Candidate C	(0.96, 0.27)	(0.65, 0.54)	(0.83, 0.46)	(0.96, 0.27)
Candidate D	(0.46, 0.73)	(0.96, 0.27)	(0.65, 0.54)	(0.83, 0.46)
Candidate E	(0.83, 0.46)	(0.83, 0.46)	(0.96, 0.27)	(0.96, 0.27)
Candidate F	(0.65, 0.54)	(0.46, 0.73)	(0.83, 0.46)	(0.65, 0.54)
Candidate G	(0.96, 0.27)	(0.65, 0.54)	(0.65, 0.54)	(0.83, 0.46)

The second stage in the PF-MUNRA method is to normalize the decision matrix using three different approaches, namely linear, vector, and non-linear, each of which has characteristics in equalizing the data scale so that it can be fairly compared across criteria. Linear normalization is carried out by converting the alternative values into a certain range based on the maximum or minimum values, thereby maintaining the proportion of the comparison directly calculated using (4) and (5);

$$r_{11}^{\mu} = \frac{x_{11}^{\mu}}{\max x_{i1}^{\mu}} = \frac{0.83}{0.96} = 0.865$$

$$r_{11}^v = \frac{x_{11}^v}{\max x_{i1}^v} = \frac{0.46}{0.73} = 0.630$$

The results of the linear normalization in the PF-MUNRA method are presented in Table 5.

Table 5. Linear Normalization of the PF-MUNRA Method

Alternative	C1	C2	C3	C4
Candidate A	(0.865, 0.630)	(1.000, 0.370)	(0.865, 0.852)	(0.677, 1.000)
Candidate B	(0.677, 0.740)	(0.865, 0.630)	(1.000, 0.500)	(0.865, 0.852)
Candidate C	(1.000, 0.370)	(0.677, 0.740)	(0.865, 0.852)	(1.000, 0.500)
Candidate D	(0.479, 1.000)	(1.000, 0.370)	(0.677, 1.000)	(0.865, 0.852)
Candidate E	(0.865, 0.630)	(0.865, 0.630)	(1.000, 0.500)	(1.000, 0.500)
Candidate F	(0.677, 0.740)	(0.479, 1.000)	(0.865, 0.852)	(0.677, 1.000)
Candidate G	(1.000, 0.370)	(0.677, 0.740)	(0.677, 1.000)	(0.865, 0.852)

Vector normalization uses the concept of vector length, where each value is divided by the square root of the sum of the squares of all values in that criterion, thereby producing a scale based on proportions relative to the entire data calculated using (6) and (7).

$$y_{11}^{\mu} = \frac{x_{11}^{\mu}}{\sqrt{\sum_{i=1}^7 (x_{i1}^{\mu})^2}} = \frac{0.83}{\sqrt{4.2776}} = \frac{0.83}{2.068} = 0.401$$

$$y_{11}^v = \frac{x_{11}^v}{\sqrt{\sum_{i=1}^7 (x_{i1}^v)^2}} = \frac{0.46}{\sqrt{1.685}} = \frac{0.46}{1.298} = 0.345$$

The results of the vector normalization in the PF-MUNRA method are presented in Table 6.

Table 6. Vector Normalization of the PF-MUNRA Method

Alternative	C1	C2	C3	C4
Candidate A	(0.401, 0.354)	(0.464, 0.208)	(0.381, 0.394)	(0.298, 0.462)
Candidate B	(0.314, 0.416)	(0.401, 0.354)	(0.440, 0.231)	(0.381, 0.394)
Candidate C	(0.464, 0.208)	(0.314, 0.416)	(0.381, 0.394)	(0.440, 0.231)
Candidate D	(0.222, 0.562)	(0.464, 0.208)	(0.298, 0.462)	(0.381, 0.394)
Candidate E	(0.401, 0.354)	(0.401, 0.354)	(0.440, 0.231)	(0.440, 0.231)

Candidate F	(0.314, 0.416)	(0.222, 0.562)	(0.381, 0.394)	(0.298, 0.462)
Candidate G	(0.464, 0.208)	(0.314, 0.416)	(0.298, 0.462)	(0.381, 0.394)

Meanwhile, non-linear normalization applies a specific transformation function to adjust the data distribution, thus being more capable of handling extreme differences or uneven value variations, calculated using (8) and (9).

$$p_{11}^{\mu} = \left(\frac{x_{11}^{\mu}}{\max x_{i1}^{\mu}} \right)^2 = \left(\frac{0.83}{0.96} \right)^2 = 0.748$$

$$p_{11}^{\nu} = \left(\frac{x_{11}^{\nu}}{\max x_{i1}^{\nu}} \right)^2 = \left(\frac{0.46}{0.73} \right)^2 = 0.397$$

The results of the non-linear normalization in the PF-MUNRA method are presented in Table 7.

Table 7. Non-linear normalization of the PF-MUNRA method.

Alternative	C1	C2	C3	C4
Candidate A	(0.748, 0.397)	(1.000, 0.137)	(0.748, 0.726)	(0.458, 1.000)
Candidate B	(0.458, 0.547)	(0.748, 0.397)	(1.000, 0.250)	(0.748, 0.726)
Candidate C	(1.000, 0.137)	(0.458, 0.547)	(0.748, 0.726)	(1.000, 0.250)
Candidate D	(0.230, 1.000)	(1.000, 0.137)	(0.458, 1.000)	(0.748, 0.726)
Candidate E	(0.748, 0.397)	(0.748, 0.397)	(1.000, 0.250)	(1.000, 0.250)
Candidate F	(0.458, 0.547)	(0.230, 1.000)	(0.748, 0.726)	(0.458, 1.000)
Candidate G	(1.000, 0.137)	(0.458, 0.547)	(0.458, 1.000)	(0.748, 0.726)

The third stage in the PF-MUNRA method is to combine the normalization results by considering the weight of each criterion. These weights reflect the relative importance of each criterion in the decision-making process. The aggregate values are calculated using (10), (11), and (12).

$$r_1' = \sum_{j=1}^4 w_j * (r_{1j}^{\mu} - r_{1j}^{\nu}) = 0.081 + 0.217 + 0.002 + (-0.050) = 0.250$$

$$y_1' = \sum_{j=1}^4 w_j * (y_{1j}^{\mu} - y_{1j}^{\nu}) = 0.016 + 0.088 + (-0.002) + (-0.025) = 0.077$$

$$p_1' = \sum_{j=1}^4 w_j * (p_{1j}^{\mu} - p_{1j}^{\nu}) = 0.121 + 0.298 + 0.003 + (-0.084) = 0.338$$

The results of the weighted normalization values in the PF-MUNRA method are presented in Table 8.

Table 8. Weighted normalization of the PF-MUNRA method.

Alternative	r_i'	y_i'	p_i'
Candidate A	0.250	0.077	0.338
Candidate B	0.139	0.011	0.210
Candidate C	0.275	0.084	0.387
Candidate D	-0.010	-0.056	-0.048
Candidate E	0.317	0.097	0.474
Candidate F	-0.249	-0.180	-0.377
Candidate G	0.148	0.026	0.187

The final stage of the PF-MUNRA method is to integrate all aggregation results into a single final value using the basic MUNRA concept, which is calculated using equation (13).

$$S_1 = \alpha r_1' + \beta y_1' + \gamma p_1' = (0.5 * 0.250) + (0.5 * 0.077) + (0.5 * 0.338) = 0.333$$

The results of the final value calculation in the PF-MUNRA method are presented in Table 9.

Table 9. Final value of the PF-MUNRA method.

Alternative	Final Score
Candidate A	0.333
Candidate B	0.180
Candidate C	0.373
Candidate D	-0.058
Candidate E	0.444
Candidate F	-0.403
Candidate G	0.180

The final result of applying the PF-MUNRA method is the preference value for each alternative, reflecting the relative performance level based on all the criteria that have been considered. These values are then used as the basis for the ranking process, where the alternative with the highest preference value is considered the best choice because it has the most optimal suitability to the criteria used. This process allows decision-making to be carried out more objectively and structurally, taking into account both quantitative aspects and uncertainties represented through Pythagorean fuzzy. All the alternative ranking results obtained from the PF-MUNRA calculations are presented in full in Table 10.

Table 10. PF-MUNRA ranking results.

Alternative	Final Score	Rank
Candidate E	0.444	1
Candidate C	0.373	2
Candidate A	0.333	3
Candidate B	0.18	4
Candidate G	0.18	5
Candidate D	-0.058	6
Candidate F	-0.403	7

The ranking results show that Candidate E obtained the highest preference score of 0.444, thus occupying the first rank and is considered the best alternative based on the PF-MUNRA method. The next positions are occupied by Candidate C with a score of 0.373 and Candidate A with a score of 0.333, indicating relatively good performance compared to the other alternatives. Candidate B and Candidate G have the same score of 0.18, yet they are still distinguished in the ranking order, followed by Candidate D with a negative score of -0.058, indicating less than optimal performance. Meanwhile, Candidate F is in the last position with a score of -0.403, indicating that this alternative has the lowest level of performance compared to the other candidates based on all evaluated criteria.

Analysis and Evaluation Result

Analysis and evaluation of results constitute an important stage aimed at interpreting the calculation outcomes obtained from the implementation of the proposed method, as well as assessing the consistency, effectiveness, and reliability of the decisions produced. At this stage, the ranking results of alternatives are examined in depth to identify differences in preference values and to understand how each criterion contributes to the final ranking position of every alternative. This analysis provides a clearer understanding of the decision-making patterns generated by the model and helps determine whether the results align with the expected objectives and real-world conditions. Furthermore, the evaluation process can be strengthened through comparative analysis with other decision-

making methods or by conducting sensitivity analysis to measure the stability of the ranking results against changes in parameter values, criterion weights, or input data. Such evaluations are essential to ensure that the proposed method produces robust and consistent decisions under different scenarios. This stage therefore plays a significant role in validating that the resulting decisions are not only mathematically accurate and systematic, but also practical, reliable, and accountable for supporting decision-making processes in complex environments.

The comparison between the original ranking and the ranking produced by the PF-MUNRA method is conducted to analyze the extent of changes in the positions of alternatives after undergoing data processing based on Pythagorean fuzzy. This stage aims to observe the conformity and differences in ranking patterns without directly emphasizing the final results, but rather to understand the influence of the method on the structure of alternative evaluations. With this comparison, an overview can be obtained of how the processes of normalization, weighting, and aggregation in PF-MUNRA affect the order of alternatives compared to the initial ranking. The results of the comparison between the original ranking and PF-MUNRA are presented in Figure 2.

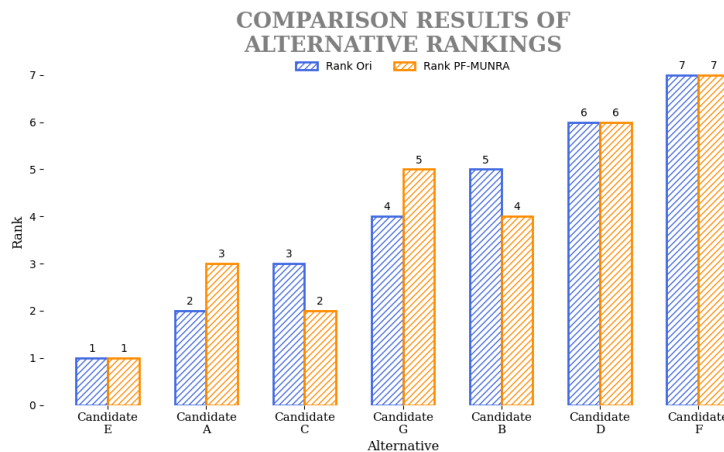


Figure 2. Results of Alternative Ranking Comparison

The comparison of the rankings of alternatives between the initial Rank Ori and the PF-MUNRA method shows that some alternatives maintain their ranking positions, while others experience changes in order between the two methods. Candidate E, Candidate D, and Candidate F demonstrate consistency as they have the same ranking in both approaches, whereas Candidate A and Candidate G experience a drop in ranking in PF-MUNRA compared to the initial ranking. Conversely, Candidate C and Candidate B show an improvement in position in the PF-MUNRA results. This visualization provides a clear depiction of the differences in ranking distribution between methods, as well as showing how the PF-MUNRA approach affects the final evaluation of each alternative. The comparative ranking results show that Candidate E maintains the highest position in both the initial method and PF-MUNRA because it demonstrates the most consistent and superior performance on the dominant criteria that carry high weight in the evaluation process. The stability of this position indicates that Candidate E's scores are not only strong in one particular aspect, but also balanced across most assessment criteria. The integration of Pythagorean Fuzzy logic allows for a more flexible representation of membership and non-membership degrees in handling the uncertainty of assessment data, making the evaluation more realistic and adaptive. The combination of these factors causes some candidates to experience changes in ranking positions, yet Candidate E remains at the top rank due to its consistently superior level even after considering the effects of multiple normalization and fuzzy uncertainty representation.

Overall, the analysis and evaluation stage show that the application of the PF-MUNRA method is able to provide a more comprehensive understanding of the performance of each alternative through a systematic and structured data processing process. The comparison between the initial ranking and the results of the method indicates a dynamic change in positions that reflects the sensitivity of the method to variations in values and criteria weights, thus not solely relying on the initial assessment. This confirms that the Pythagorean fuzzy-based approach has advantages in accommodating uncertainty and enhancing objectivity in the evaluation process. In addition, consistency across several alternatives also indicates that this method maintains result stability under certain conditions, making it reliable in supporting decision-making. Thus, the results obtained not only provide a ranking of the best alternatives but also strengthen the validity of decisions through deeper analysis, so the PF-MUNRA method can be considered an effective and reliable approach in solving multi-criteria decision-making problems.

Evaluation using a sensitivity test of criterion weights is carried out to measure the level of stability and resilience of the method against changes in preferences in the decision-making process. This test aims to determine the extent to which small variations in criterion weights can affect the final values and the ranking of alternatives produced. By systematically conducting scenarios of weight changes, both increases and decreases, it can be analyzed whether the method used is robust or rather sensitive to these changes. Through this approach, not only can the consistency of results be tested, but it is also possible to identify which criteria are most influential in determining decisions, thereby providing a deeper understanding of the characteristics of the developed model.

Sensitivity testing was conducted by systematically applying scenarios of changes in criterion weights through additions and reductions of ± 0.1 and ± 0.05 from the initial weight values. Each change was applied alternately to each criterion while maintaining the total weight proportion to remain consistent. This approach aims to test the extent to which variations in weights on a small to moderate scale can affect the final values and the ranking of the alternatives generated. By comparing the results of each scenario with the initial conditions, the level of stability and resilience of the method to changes in preferences can be analyzed, as well as identifying the criteria that have the most significant influence in the decision-making process. All results of these sensitivity testing scenarios are presented in full in Figure 3 as a basis for further analysis of the method's consistency and robustness.

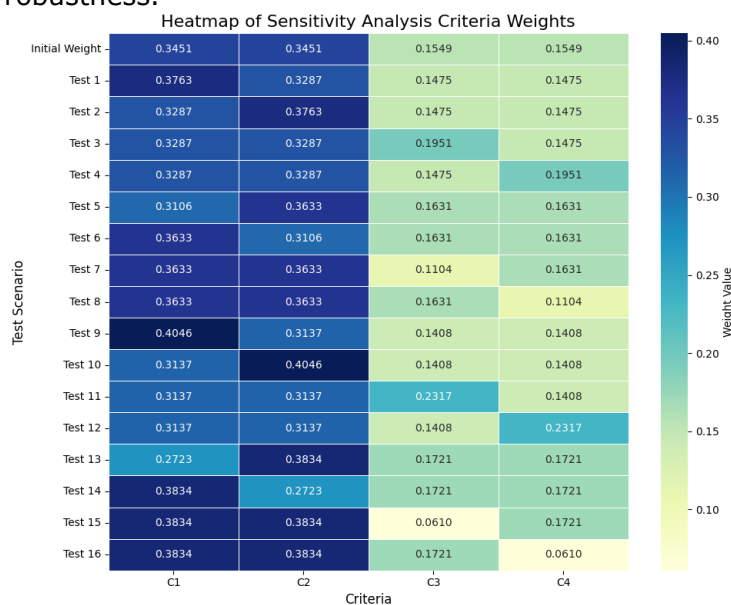


Figure 3. Sensitivity Test of Criteria Weights

The sensitivity test results that have been carried out through various scenarios of changes in criteria weights, both with increases and decreases of ± 0.1 and ± 0.05 , were subsequently used in each scenario to recalculate the final values and comprehensively rank the alternatives. This process aims to evaluate whether weight changes have a significant impact on the position of each alternative within the ranking structure. By comparing the ranking results in each scenario with the initial condition, the level of stability and consistency of the method in facing preference variations can be identified. All of these ranking results are then visualized in the form of a chart in Figure 4, making it easier to observe the pattern of ranking changes, identify alternatives that are stable or sensitive to weight changes, and assess the extent to which the method used can maintain the reliability of results under various testing conditions.

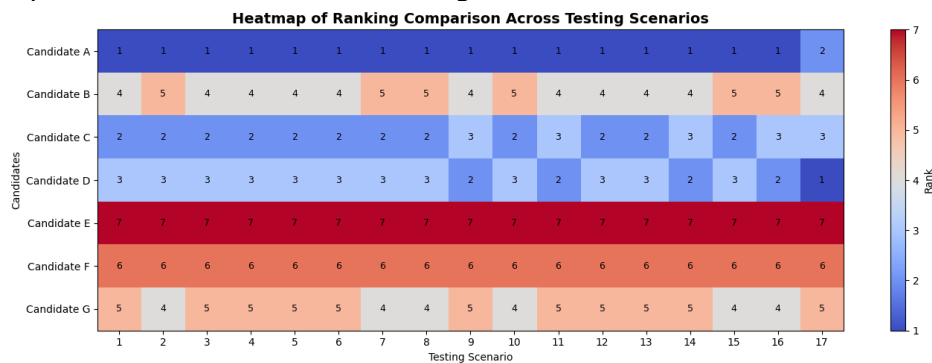


Figure 4. Ranking Results from Sensitivity Analysis

The comparison chart of ranking results based on various testing scenarios shows that most alternatives have a high level of stability against changes in criteria weights. It can be seen that some candidates maintain relatively constant positions across scenarios, such as candidates consistently ranking at the top or bottom, indicating that the performance of these alternatives is not greatly affected by weight variations. On the other hand, there are some candidates that experience ranking fluctuations in certain scenarios, although these changes are not too drastic and still remain within a close positional range. This pattern indicates that the method used has fairly good sensitivity in distinguishing alternatives, while still being able to maintain overall result consistency. The results of this graph show that the applied approach has a balance between stability and responsiveness to changes, making it reliable in producing robust decisions under various test conditions.

Spearman's rank correlation is used to analyze the level of relationship and consistency of the ranks of alternatives resulting from changes in criteria weights in the PF-MUNRA method. This test aims to determine the extent to which changes in weight distribution affect the stability of alternative ranking results. By comparing the initial rankings with the rankings after weight change scenarios are applied, the Spearman coefficient can indicate whether the order of alternatives remains consistent or undergoes significant changes. A correlation value close to 1 indicates that the PF-MUNRA method has a high level of stability and reliability against variations in criteria weights, while a lower value indicates the method's sensitivity to changes in decision-makers' preferences. The results of the Spearman correlation test in each scenario of weight changes showed that the correlation values remained in the very strong category, indicating that the PF-MUNRA method is able to maintain the consistency of alternative rankings even when there are changes in criteria weights. Overall, the Spearman correlation values from each weight change test are presented in Figure 1.

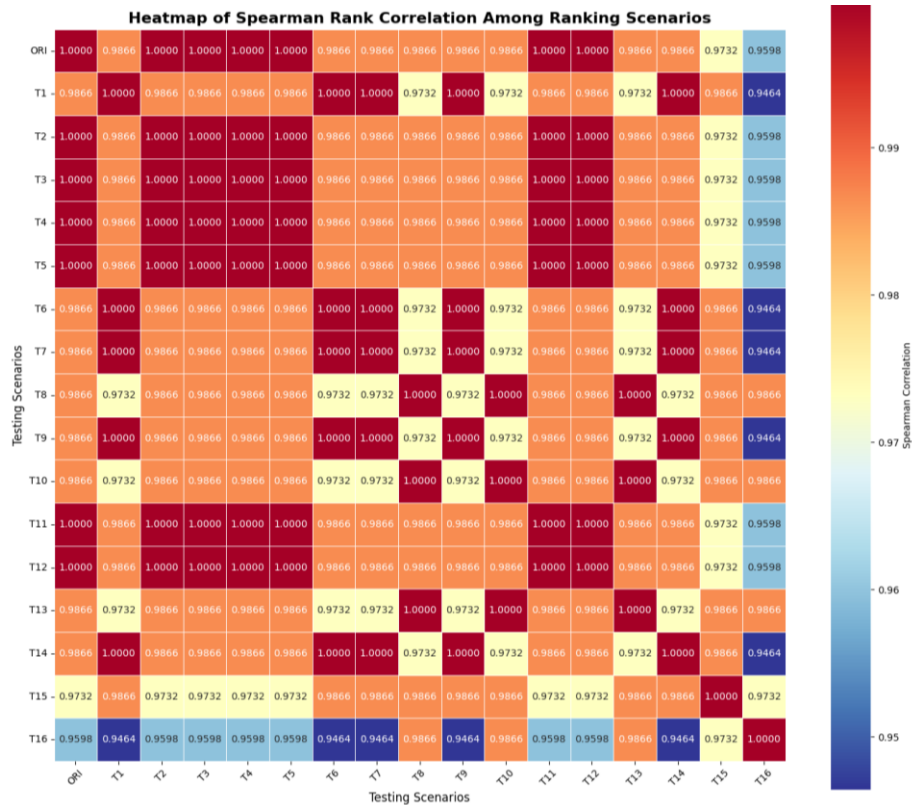


Figure 5. Spearman Correlation Result Among Ranking Scenarios

The results of the Spearman rank correlation show that the PF-MUNRA method has a very high level of consistency and stability against changes in criteria weights across all testing scenarios. The correlation values obtained range from 0.9464 to 1.0000, indicating a very strong relationship between the ranks of alternatives in each scenario of weight changes. Certain tests such as ORI with T2, T3, T4, T5, T11, and T12 resulted in a perfect correlation value of 1.0000, indicating that changes in weights do not affect the ranking order of alternatives. Meanwhile, the lowest correlation value of 0.9464 occurred in the relationship between T16 and several other scenarios, but this value is still considered very strong, meaning that the ranking changes that occurred are relatively small. In addition, most testing scenarios produced correlation values of 0.9732 and 0.9866, which further strengthens that the PF-MUNRA method can maintain the stability of ranking results even when criterion weights are varied gradually. Based on Spearman correlation test results, the correlation value range obtained was between 0.9464 and 1.0000. Referring to the correlation interpretation category, this falls into the very strong category. This indicates that changes in criterion weights in the PF-MUNRA method do not have a significant impact on changes in alternative rankings. Thus, the PF-MUNRA method has a very high level of consistency, stability, and robustness in generating decisions, making it reliable to use in multi-criteria decision-making processes even when there are variations in criterion weights.

4. CONCLUSION

The implementation of the PF-MUNRA method is capable of providing effective solutions in multi-criteria decision-making by considering uncertainty through a Pythagorean fuzzy approach. This method successfully processes assessment data into structured preference values, thereby producing alternative rankings that are more

objective and systematic. In addition, the integration of criteria weighting and the normalization process used allows for a more comprehensive evaluation of each alternative. The analysis results also show that this method can provide different perspectives compared to initial rankings, thus potentially increasing the quality of the decisions made. Therefore, PF-MUNRA can be considered a reliable and relevant approach to be applied in various multi-criteria decision-making problems. The sensitivity analysis results indicate that the PF-MUNRA method has a high level of stability and robustness against changes in criteria weights, both in scenarios of increase and decrease. The changes in weights applied do not cause significant shifts in the ranking structure of alternatives, so the positions of each alternative tend to remain consistent across various testing conditions. This demonstrates that PF-MUNRA is not sensitive to small fluctuations in criteria weights and can maintain the consistency of evaluation results. For future work, further research can explore the integration of PF-MUNRA with other advanced weighting and optimization techniques to enhance decision accuracy and adaptability in more complex environments. In addition, the method can be tested on larger-scale datasets and various real-world application domains to evaluate its scalability and generalization capabilities. Future studies may also compare PF-MUNRA with other fuzzy-based and hybrid multi-criteria decision-making methods to further validate its effectiveness and computational performance.

5. REFERENCES

- [1] I. Corbo, F. Favieri, G. Forte, and M. Casagrande, "Decision-making under uncertainty in healthy and cognitively impaired aging: A systematic review and meta-analysis," *Arch. Gerontol. Geriatr.*, vol. 129, p. 105643, 2025, doi: 10.1016/j.archger.2024.105643.
- [2] R. Alekperov, V. Salahli, and R. Imamguluyev, "Decision Making Under Uncertainty: A Z-Number-Based Regret Principle," *Mathematics*, vol. 13, no. 22. p. 3579, 2025. doi: 10.3390/math13223579.
- [3] T. Badings, T. D. Simão, M. Suilen, and N. Jansen, "Decision-making under uncertainty: beyond probabilities: Challenges and perspectives," *Int. J. Softw. Tools Technol. Transf.*, vol. 25, no. 3, pp. 375–391, May 2023, doi: 10.1007/s10009-023-00704-3.
- [4] H. Tissot, "FormulAI: Designing Rule-Based Datasets for Interpretable and Challenging Machine Learning Tasks," *Artif. Intell. Appl.*, vol. 3, no. 1 SE-Research Article, pp. 72–82, Mar. 2024, doi: 10.47852/bonviewAIA42021781.
- [5] İ. Otay, S. Çevik Onar, B. Öztayşi, and C. Kahraman, "Evaluation of sustainable energy systems in smart cities using a Multi-Expert Pythagorean fuzzy BWM & TOPSIS methodology," *Expert Syst. Appl.*, vol. 250, p. 123874, 2024, doi: <https://doi.org/10.1016/j.eswa.2024.123874>.
- [6] S. Zhang and M. O. Esangbedo, "Urban Scenic Spot Activity Center Investment: Strategic Construction Company Selection Using the Grey System-II Thinking Compromise Ranking of Alternatives from Distance to Ideal Solution Multi-Criteria Decision-Making Method," *Systems*, vol. 13, no. 1. 2025. doi: 10.3390/systems13010067.
- [7] P. Zandi, M. Ajalli, and N. S. Ekhtiyati, "An extended simple additive weighting decision support system with application in the food industry," *Decis. Anal. J.*, vol. 14, p. 100553, 2025, doi: <https://doi.org/10.1016/j.dajour.2025.100553>.
- [8] Y. Tang, X. Zhang, Y. Zhou, Y. Huang, and D. Zhou, "A new correlation belief function in Dempster-Shafer evidence theory and its application in classification," *Sci. Rep.*, vol. 13, no. 1, p. 7609, May 2023, doi: 10.1038/s41598-023-34577-y.
- [9] L. Liu, X. Wan, J. Li, W. Wang, and Z. Gao, "An Improved Entropy-Weighted Topsis Method for Decision-Level Fusion Evaluation System of Multi-Source Data," *Sensors*,

- vol. 22, no. 17. 2022. doi: 10.3390/s22176391.
- [10] Z. Li, Y. Wang, J. Xie, Y. Cheng, and L. Shi, "Hybrid multi-criteria decision-making evaluation of multiple renewable energy systems considering the hysteresis band principle," *Int. J. Hydrogen Energy*, vol. 49, pp. 450–462, 2024, doi: <https://doi.org/10.1016/j.ijhydene.2023.09.059>.
- [11] S. DüNDAR, "Performance evaluation of IPARD-II rural development programs with integrated DIBR-RAWEC methods," *Pamukkale Üniversitesi Mühendislik Bilim. Derg.*, vol. 31, no. 3, pp. 339–350, 2025, [Online]. Available: <https://dergipark.org.tr/en/pub/pajes/article/1727829>
- [12] Sait Gül and Ali Aydoğdu, "Novel Entropy Measure Definitions and Their Uses in a Modified Combinative Distance-Based Assessment (CODAS) Method Under Picture Fuzzy Environment," *Informatica*, vol. 32, no. 4, pp. 759–794, Aug. 2021, doi: 10.15388/21-INFOR458.
- [13] J. Liu, X. Bao, and L. Chen, "Artificial intelligence in educational technology and transformative approaches to English language using fuzzy framework with CRITIC-TOPSIS method," *Sci. Rep.*, vol. 15, no. 1, p. 25542, 2025, doi: 10.1038/s41598-025-09844-9.
- [14] E. Bozdogan and C. Kadaifci, "A Distance-Based Approach to Fuzzy Cognitive Maps Using Pythagorean Fuzzy Sets," *Int. J. Fuzzy Syst.*, vol. 27, no. 1, pp. 93–109, 2025, doi: 10.1007/s40815-024-01766-4.
- [15] H. D. Arora and A. Naithani, "On some new fuzzy entropy measure of Pythagorean fuzzy sets for decision-making based on an extended TOPSIS approach," *J. Manag. Anal.*, vol. 11, no. 1, pp. 87–109, Jan. 2024, doi: 10.1080/23270012.2024.2301748.
- [16] N. Ersoy and S. Taslak, "Comparative Analysis of MCDM Methods for the Assessment of Corporate Sustainability Performance in Energy Sector," *Ege Acad. Rev.*, vol. 23, no. 3, pp. 341–362, 2023, doi: 10.21121/eab.986122.
- [17] K. Yu, Q. Wu, X. Chen, W. Wang, and A. Mardani, "An integrated MCDM framework for evaluating the environmental, social, and governance (ESG) sustainable business performance," *Ann. Oper. Res.*, vol. 342, no. 1, pp. 987–1018, 2024, doi: 10.1007/s10479-023-05616-8.
- [18] M. J. Khan, J. C. R. Alcantud, W. Kumam, P. Kumam, and N. A. Alreshidi, "Expanding Pythagorean fuzzy sets with distinctive radii: disc Pythagorean fuzzy sets," *Complex Intell. Syst.*, vol. 9, no. 6, pp. 7037–7054, 2023, doi: 10.1007/s40747-023-01062-y.
- [19] M. C. Bozyigit, M. Olgun, and M. Ünver, "Circular Pythagorean Fuzzy Sets and Applications to Multi-Criteria Decision Making," *Informatica*, vol. 34, no. 4, pp. 713–742, Sep. 2023, doi: 10.15388/23-INFOR529.
- [20] R. Kumar, N. Gandotra, and Suman, "A novel pythagorean fuzzy entropy measure using MCDM application in preference of the advertising company with TOPSIS approach," *Mater. Today Proc.*, vol. 80, pp. 1742–1746, 2023, doi: <https://doi.org/10.1016/j.matpr.2021.05.497>.
- [21] M. Baydaş and D. Pamučar, "Determining Objective Characteristics of MCDM Methods under Uncertainty: An Exploration Study with Financial Data," *Mathematics*, vol. 10, no. 7. p. 1115, 2022. doi: 10.3390/math10071115.
- [22] H. Li and M. Yazdi, "An Advanced TOPSIS-PFS Method to Improve Human System Reliability BT - Advanced Decision-Making Methods and Applications in System Safety and Reliability Problems: Approaches, Case Studies, Multi-criteria Decision-Making, Multi-objective Decision-Making, Fuzzy Risk-Based Models," H. Li and M. Yazdi, Eds. Cham: Springer International Publishing, 2022, pp. 109–125. doi: 10.1007/978-3-031-07430-1_7.
- [23] A. Ulutaş, F. Ecer, and Z. Turskis, "Multiple Normalization Rating Analysis (MUNRA) and its application to digital supplier selection in the textile industry," *Technol. Econ.*

- Dev. Econ.*, vol. 31, no. 6 SE-Articles, pp. 2074–2104, doi: 10.3846/tede.2025.25346.
- [24] M. A. M. Al-Gerafi *et al.*, "Promoting inclusivity in education amid the post-COVID-19 challenges: An interval-valued fuzzy model for pedagogy method selection," *Int. J. Manag. Educ.*, vol. 22, no. 3, p. 101018, 2024, doi: <https://doi.org/10.1016/j.ijme.2024.101018>.
- [25] M. A. Al-Gerafi *et al.*, "Designing of an effective e-learning website using inter-valued fuzzy hybrid MCDM concept: A pedagogical approach," *Alexandria Eng. J.*, vol. 97, pp. 61–87, 2024, doi: <https://doi.org/10.1016/j.aej.2024.04.012>.
- [26] R. Singh *et al.*, "A historical review and analysis on MOORA and its fuzzy extensions for different applications," *Heliyon*, vol. 10, no. 3, p. e25453, 2024, doi: <https://doi.org/10.1016/j.heliyon.2024.e25453>.
- [27] A. Ulutaş, A. Topal, and F. Ecer, "The Alternative Prioritization and Assessment System (ALPAS) Method for Environmental Performance Evaluation," *Mathematics*, vol. 13, no. 20, p. 3333, 2025. doi: 10.3390/math13203333.
- [28] F. Mizrak, K. C. Mizrak, and G. R. Akkartal, "Developing a strategic framework for airline destination selection: A multi-criteria decision-making approach applied to Turkish airlines," *Transp. Res. Interdiscip. Perspect.*, vol. 29, p. 101322, 2025, doi: <https://doi.org/10.1016/j.trip.2025.101322>.
- [29] A. Yudhistira, "Penerapan Metode Simple Additive Weighting dan Pembobotan Entropy Untuk Penentuan Teknisi Terbaik," *J. Artif. Intell. Technol. Inf.*, vol. 2, no. 3 SE-Articles, pp. 143–152, Sep. 2024, doi: 10.58602/jaiti.v2i3.133.
- [30] M. A. Alsalem *et al.*, "Evaluation of trustworthy artificial intelligent healthcare applications using multi-criteria decision-making approach," *Expert Syst. Appl.*, vol. 246, p. 123066, 2024, doi: <https://doi.org/10.1016/j.eswa.2023.123066>.